MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

PEM0036 – CALCULUS

(Foundation in Engineering)

6 OCTOBER 2015
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5) pages** including cover page and appendix with **FOUR (4) questions** only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.
4. Only non-programmable calculator is allowed.

QUESTION 1 [25 marks]

(a) Find:

(i) $\lim_{x \to C}(5x+1)$. (1 mark)

(ii) $\lim_{x \to \infty}\left[\left(\frac{2}{3}\right)^x + 4\right]$ (3 marks)

(b) Show that $\lim_{x \to -4} \frac{2-\sqrt{x^2-12}}{4+x}$ can be obtained by using both simplification method and L'Hopital's Rule. (6 marks)

(c) Refer to **Figure 1**, determine $\lim_{x \to 0} g(x)$. (4 marks)

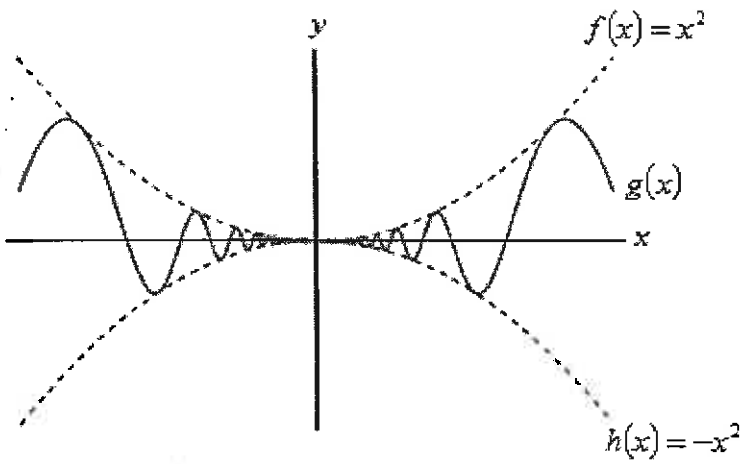

Figure 1

(d) Check whether $\lim_{t \to -2} f(t)$ exists if $f(t) = \begin{cases} t^2+5 & t < -2 \\ -3t+3 & t \geq -2 \end{cases}$ (5 marks)

(e) Find the asymptotes (horizontal/vertical/slant) for the following functions:

(i) $f(x) = \frac{x^2-3x}{2x-x^2}$ (3 marks)

(ii) $f(x) = \frac{2}{3x^2}$ (3 marks)

Continued.....

QUESTION 2 [25 marks]

(a) Differentiate $\frac{y}{2x} = e^{-8xy^2}$ with respect to x. (4 marks)

(b) Find the extreme values of function $y = 2x\sqrt{9-x^2}$ and where they occur. (10 marks)

(c) A certain curve's derivative is given as $f'(x) = 2x^3 + 2x^2 - 12x$. Do the following:

(i) Determine where the inflection point(s) occur and interval of concavity. Round up any fractions/roots up to 3 decimals throughout the computation. (6 marks)

(ii) Use second derivative to determine whether the curve has local minimum or maximum point(s). (5 marks)

QUESTION 3 [25 marks]

(a) Determine the area of the region bounded by $\sqrt{x}$, xe^{-x^2}, $x = 2$ and the y-axis where $\sqrt{x} \geq xe^{-x^2}$. You may write your answer in 3 decimals. (6 marks)

(b) Determine the volume of the solid obtained by rotating the portion of the region bounded by $f(x) = \cos\left(\frac{1}{2}x\right)$ and $g(x) = \sin\left(\frac{1}{2}x\right)$ as shaded in **Figure 2** which lies in the first quadrant about the x-axis. (7 marks)

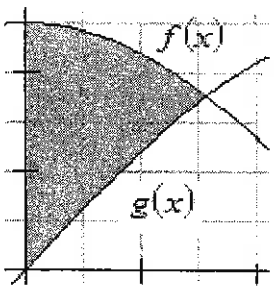

Figure 2

(c) Determine the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = \frac{1}{8}x + 2$ and $x = 0$ about the line $y = 5$. Sketch the curves and shade the bounded region to aid your computation. Round up any fractions/roots up to 3 decimals throughout the computation. (12 marks)

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QUESTION 4 [25 marks]

(a) Verify whether $y = e^{-x} + Ce^{-\frac{7}{2}x}$ is the solution of the differential equation $2y'+7y = 5e^{-x}$. (3 marks)

(b) Solve the following differential equations:

(i) $x^{\frac{1}{2}} \frac{dy}{dx} = e^{8y+\sqrt{x}}$ (5 marks)

(ii) $x\frac{dy}{dx} + 2y = 8 - \frac{1}{x}$

(Express your answer as an explicit equation) (7 marks)

(iii) $y''+9y'+20y = 0$; $y(0) = -4$; $y'(0) = 19$ (10 marks)

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APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[e^x] = e^x \qquad \frac{d}{dx}[\ln x] = \frac{1}{x} \; ; \quad x > 1$$

$$\frac{d}{dx}[sin^{-1}x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[cos^{-1}x] = -\frac{1}{\sqrt{1-x^2}} \; for -1 < x < 1$$

$$\frac{d}{dx}[tan^{-1}x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[cot^{-1}x] = -\frac{1}{1+x^2} \; for -\infty < x < \infty$$

$$\frac{d}{dx}[sec^{-1}x] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[csc^{-1}x] = -\frac{1}{|x|\sqrt{x^2-1}} \; for |x| > 1$$

$$Area = \int_a^b [f(x) - g(x)]\, dx$$

$$Volume\ (Disk) = \pi \int_a^b [f(x)]^2\, dx$$

$$Volume\ (Washer) = \pi \int_a^b [f(x)]^2 - [g(x)]^2\, dx$$

$$Volume\ (\ Cylindrical\ Shells) = \int_a^b 2\pi(shell\ radius)(shell\ height)\, dx$$

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